

HYDRODYNAMICS AND MASS TRANSFER IN TURBULENT LIQUID FILM FLOW INVOLVING THE INLET PORTION

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Relationships for mass transfer coefficient in turbulent liquid film flow involving the inlet section have been derived theoretically. It was found that previously published experimental results were well explained by this theory.

Recently, the inlet portion film flow is of the growing interest¹⁻¹⁴. The results of accelerated film flow over vertical and inclined surfaces can be used in studying the hydrodynamics and mass transfer in packed columns where a liquid film flows over the packing material, though those of the inlet portion hydrodynamics and mass transfer can be used directly for heat and mass transfer apparatus.

Various solution methods with various degrees of accuracy have been used in studying the inlet portion liquid film hydrodynamics. The most comprehensive study has been made⁴. The inlet portion liquid film hydrodynamics and mass transfer are investigated by the method suggested in¹⁴.

A system of equations describing the turbulent liquid film flow consists of the Navier-Stokes equations and that of convective diffusion and may be written in the form¹⁵:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \sin \alpha_1 - \frac{1}{\varrho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(v_{ef} \frac{\partial u}{\partial y} \right) \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{\partial}{\partial y} \left(D_{ef} \frac{\partial c}{\partial y} \right) \quad (2)$$

at

$$y = 0, \quad u = v = 0, \quad c = 0$$

at

$$y = \delta(x), \quad \partial u / \partial y = 0, \quad c = C_1, \quad (3)$$

where $v_{ef} = v + v_t$, $D_{ef} = D + D_t$ and v , D , v_t , D_t are the transport coefficients for the molecular and turbulent liquid film flow conditions, respectively, α_1 is the wetted channel slope angle.

The fact that the mean film thickness $\delta(x)$ which varies with the distance from the inlet is obtained from a solution makes the solution of the problem (1)–(3) difficult.

If $v_t = D_t = 0$, then $v_{ef} = v$, $D_{ef} = D$. In this case the system of Eqs (1)–(3) along with the boundary conditions (3) describes mass transfer into the inlet portion laminar liquid film.

We shall illustrate the method suggested in¹⁴ with reference to this problem. Consider a case when mass transfer resistance is concentrated in the liquid phase; the concentration at the liquid film surface C_1 is taken to be constant. The velocities u , v and transversal coordinate y in the system of Eqs (1)–(2) can be made dimensionless in two different ways: 1) the instant velocity may be related to the velocity in the initial cross-section and the transversal coordinate – to the liquid efflux slot dimension; 2) the instant velocity may be related to the steady-state portion velocity and the transversal coordinate – to the steady-state portion film thickness.

Eqs (1)–(3) and boundary conditions (3) have been made dimensionless using the second method, *i.e.*

$$u = U_0 \bar{u} ; \quad x = \delta_p \operatorname{Re} \bar{x} , \quad y = \delta_p \bar{y} , \quad c = C_1 \bar{c} . \quad (4)$$

Eqs (1)–(3) and boundary conditions (3) as written in dimensionless variables take the form (the dashes are omitted):

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= 3 + \frac{\partial^2 u}{\partial y^2} , \\ \frac{\partial u}{\partial x} + \frac{\partial x}{\partial y} &= 0 , \end{aligned} \quad (5)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{1}{\operatorname{Pr}} \frac{\partial^2 c}{\partial y^2} , \quad (6)$$

$$\begin{aligned} y = 0 , \quad u = v = 0 , \quad c = 0 ; \quad x = 0 , \quad c = 0 ; \\ y = H(x)/\delta_p , \quad \frac{\partial u}{\partial y} = 0 , \quad c = 1 . \end{aligned} \quad (7)$$

To solve the problem we shall use the method described in¹⁴. For this purpose the film velocity profiles $u(x, y)$ and concentrations $c(x, y)$ can be presented in the form:

$$\varphi(x, y) = \sum_{j=1}^N A_j(x) \psi_j(y) , \quad (8)$$

where

$$\begin{aligned}\varphi(x, y) &= u(x, y), \quad c(x, y); \quad \psi_j(y) = U_j(y), \quad N1_j(y); \\ A_j(x) &= a_j(x), \quad a_{1j}(x)\end{aligned}$$

N is approximation number, $u_j(x)$, $N1_j(y)$ are complete systems of the functions for the velocity profiles and concentrations satisfying the boundary conditions.

Consider N surfaces $y_k(x)$. There is no liquid flow via each of these surfaces. Let $u_k(x)$, $v_k(x)$, $c_k(x)$ denote the velocity components and concentration at the $y_k(x)$ surface, respectively. Then, under the conditions of the steady flow considered in this paper the following relationship is valid:

$$v_k = u_k, \quad \partial y_k / \partial x, \quad k = 1, 2, 3, \dots, N. \quad (9)$$

Further on, from the flow rate conservation condition it follows:

$$\frac{\partial}{\partial x} \int_{y_{k-1}}^{y_k} u \partial y = 0. \quad (10)$$

After substituting a finite difference expression for the integral in relationship (10) we obtain the equation for constant flow rate lines:

$$\frac{dy_k}{dx} = F_k,$$

$$F_k = \frac{dy_{k-1}}{dx} - \frac{y_k - y_{k-1}}{u_k + u_{k-1}} \left(\frac{du_k}{dx} + \frac{du_{k-1}}{dx} \right) \quad (11)$$

Eqs (5) and (6) with allowance for conditions (9), (10) take the form:

$$u_k \frac{dy_k}{dx} = \frac{\partial^2 u_k}{\partial y^2} + 3 \quad (12)$$

$$u_k \frac{dc_k}{dx} = \frac{1}{Pr} \frac{\partial^2 c_k}{\partial y^2} \quad (13)$$

In order to calculate the right-hand sides of Eqs (11)–(13) it is necessary to have the velocity and concentration derivatives with respect to y . These can be calculated using the representation of the velocity and concentration profiles in relationship (9) in terms of the complete function $\psi_{jk}(x)$ systems. This system can be expressed by

means of different polynomials. In the present paper the Chebyshev and Legendre polynomials and those of the form:

$$u_k = \frac{j+1}{j} \left(\frac{y_k}{y_N} \right)^j - \left(\frac{y_k}{y_N} \right)^{j+1}$$

are used. The Chebyshev polynomials in the complete system of the functions u_k and $N1_k$ have been used in the form:

$$u_k(z_k) = T_{j+1}(z_k) - T_{j+1}(0) - (T_j(z_k) - T_j(0)) \left(\frac{j+1}{j} \right)^2 \quad (14)$$

$$N1_k(z_k) = T_{j+1}(z_k) + T_{j+1}(0) (2z_k^2 - z_k - 1), \quad (15)$$

where

$$z_k(x) = y_k(x)/y_N(x).$$

Note, that there is no significant difference in the calculation results obtained with the use of polynomials (14) and (15), however an accuracy required for the Chebyshev polynomials has been attained by a less number of terms. The Chebyshev polynomials have been chosen due to the fact that, first, these have the highest convergency, and second, the inaccuracies are distributed most advantageously in these polynomials, *viz.* uniformly over the whole range. Therefore all the further calculations are as a rule performed using the Chebyshev polynomials.

We require the coincidence of the velocity and concentration determined by formula (8) with $u_k(x)$ and $c_k(x)$ on the lines $y_k(x)$. Then for determining the coefficients $A_j(x)$, $a_j(x)$, $a_{1j}(x)$ for the velocity and concentration, respectively, we obtain the following system of linear algebraic equations:

$$\begin{aligned} \varphi_k(x) &= \sum_{j=1}^N A_j(x) \psi_{jk}[y_k(x)] \\ \varphi_k(x) &= u_k(x), \quad c_k(x); \quad \psi_{jk}[y_k(x)] = u_{jk}[y_k(x)], \quad N1_{jk}[y_k(x)]. \end{aligned} \quad (16)$$

After determining the value A_j from the system of Eqs (16) on the lines $y_k(x)$ we shall find the velocity and concentration derivatives with respect to y by means of formula (8).

The initial conditions should be prescribed at the cross-section $x = 0$. It is possible to preset a velocity profile and then using it to determine A_j ; it is also possible to preset A_j and to determine thereby the initial profile in a liquid film cross-section. In the present work we have preset the velocity profile at the initial cross-section, both

the constant profile and that of parabolic being used in calculations. The computational grid over the liquid film section at $x = 0$ may be preset both the variable and uniform. In the present work we have used a uniform grid of computation lines at $y_k(x) = k/(N - 1)$, where N is approximation number.

The system of Eqs (11) with boundary conditions (7) has been solved by the Runge-Kutta method with the constant and automatically variable steps. For smooth solutions a step value is of the order of 0.02, whereas in the cases when the second derivative has a considerably small parameter the computations show that the step value is of the order of 0.005. Note, that in some cases it is convenient to introduce a dimensionless variable of the type $x = \delta \operatorname{Re} Pz \bar{x}$, then in Eq. (13) the small parameter at the highest derivative vanishes.

The aforementioned algorithm has been used to compute the velocity and concentration fields at the inlet portion, the film thickness over the wetted channel length and the mass transfer coefficient of liquid film. These values have been calculated in¹⁴ using the algorithm presented with the available results of experimental data.

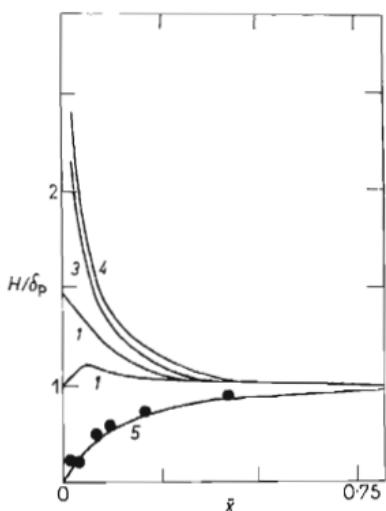


FIG. 1

Dependence of the Local Film Thickness on the Dimensionless Inlet Section Length

H/δ_p : 1 1; 2 1.5; 3 4; 4 10; 5 0.5. ● Experimental data³ for $H/\delta_p = 0.47$.

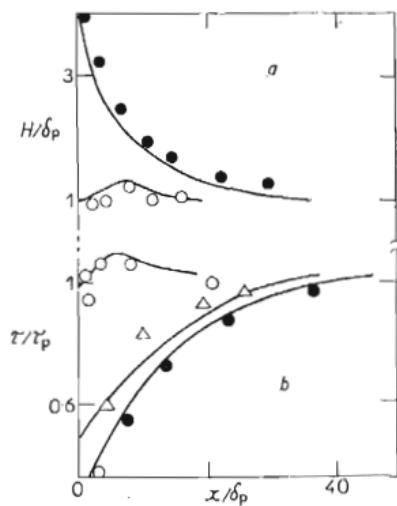


FIG. 2

Comparison of Numerical Solution (Solid Lines) with Experimental Data¹³

a Dimensionless film thickness; b dimensionless ratio of the local tangential stress to the tangential stress at the stabilization section. ● $\operatorname{Re} = q/v = 75$; $H/\delta_p = 3.7$; ○ 455; 1; △ 180; 1.3.

Fig. 1 shows as an example the relationship between the dimensionless film thickness as a structural parameter of the column-type apparatus and the inlet portion dimensionless length for different ratios of the slot width to the film thickness calculated by the Nusselt formula. The slot width has a significant effect on the accelerated film flow up to the ratio H/δ_p equal to 3. For $H/\delta_p \leq 3$ the slot width, as it is seen from Fig. 1 (curves 3, 4), does not in any way effect the accelerated film flow.

Fig. 2 shows the comparison of the calculated dimensionless film thicknesses (Fig. 2a) and dimensionless tangential stresses (Fig. 2b) for different slot dimensions with the experimental results¹³.

From Fig. 1 it follows that the stabilization portions for the liquid film and tangential stress coincide. The liquid film mass transfer has been calculated by the equality:

$$D \left(\frac{dc}{dy} \right)_{y=y_N} = \frac{d}{dx} \int_0^{y_N} uc \, dy, \quad (17)$$

where $u(x, y)$ and $c(x, y)$ are found from the solution of the system of Eqs (11)–(13) with boundary conditions (7). The mean mass transfer coefficient has been determined by the formula

$$\beta = \frac{1}{b} \int_0^{y_N} uc \, dy, \quad (18)$$

There b is certain characteristic length parameter.

For the laminar liquid film flow its value is approximated by the following expression:

$$\beta = 1.25 \frac{u_0^{0.5} D^{0.5}}{b^{0.5}} \sqrt{\left(1 + 0.65 \cdot 10^{-2} \frac{\delta_p \text{Pe}}{b} \right)}. \quad (19)$$

At $b = 1$ a formula for the inlet portion liquid film differs from the known ones in an additional coefficient $\sqrt{(1 + 0.65 \cdot 10^{-2} \delta_p \text{Pe}/b)}$ that takes into account the inlet portion mass transfer. This number has an increasing effect on the liquid phase mass transfer coefficient along with the increase in the $\text{Pe} = \text{Pr} \text{Re}$ number.

The liquid film mass transfer coefficient calculated under the inlet portion wave formation conditions has been compared in¹⁴ with that obtained from the test using the results of the mass transfer investigation under the wave formation conditions¹⁶.

In mass transfer under the turbulent inlet portion fluid flow the effective transport coefficients in the system of Eqs (1) and (2) consist of two addends, i.e. $v_{\text{ef}} = v + v_t$, $D_{\text{ef}} = D + D_t$, the turbulent transport coefficients v_t , D_t differing significantly from those of molecular v , D over the film thickness excluding the wetted channel wall and liquid–gas interface. In the present paper the transport coefficients v_t , D_t for

a liquid film are assumed to be known and vary according to a parabolic law with the maximum midmost the film, i.e.:

$$\begin{aligned} v_{ef}/v &= 1 + v_t/v = 1 + TB(\eta - \eta^2) \\ D_{ef}/D &= 1 + D_t/D = 1 + TD(\eta - \eta^2) \end{aligned} \quad (20)$$

where $TB = a\delta_p^2/v$, $TD = TB \text{Pr}$, $\text{Pr} = v/D$, $\eta = y/\delta_p$, a is proportionality factor.

As in¹⁶ too, we assume that a depends on the following parameters

$$a = a_0 f(\sigma, \varrho, v, h_0, \varepsilon, g), \quad (21)$$

where σ is the surface tension coefficient, ϱ is the liquid film density, h_0 is the mean thickness of turbulent fluid flow film, ε is dissipation energy. The functional relationship (21) can be represented in the form of the complex:

$$a = a_0 \frac{\varrho}{\sigma} \left(\frac{h_0^3 \varepsilon^3}{gv} \right)^{1/2} \quad (22)$$

where a_0 is proportionality factor.

If a_0 is known, then the problem (1)–(2) is uniquely defined. It is not possible for the present to predict beforehand the proportionality factor by means of the theory of turbulent transfer in liquid films. Therefore in the present work it is determined by approximating the experimental data drawn from^{12–22}. For solving the system (1) to (2) under boundary conditions (3) the following dimensionless constants are used:

$$u = U_0 \bar{u}, \quad y = \delta_p \bar{y}; \quad c = C_1 \bar{c}, \quad x = \delta_p \text{Re} \bar{x}, \quad P = P^0 + PU_0^2 \bar{P}, \quad (23)$$

where U_0 is the mean velocity of turbulent liquid film flow, $\text{Re} = \delta_p u_0/v$.

The equality $\partial u/\partial y = 0$ in (3) means the absence of interaction between the liquid film under turbulent flow and the gas flow. The system (1) and (2) has been solved using the method described above. We shall present certain results of the mass transfer calculation in a turbulent liquid film. The mean coefficient of liquid film mass transfer is determined by the formula:

$$\beta_1 = \frac{1}{b} \int_0^{y_N} uc \, dy = \frac{U_0 \delta_p}{b} \int_0^{y_N} uc \, dy = \frac{U_0 \delta_p}{b} \Pi R \Big|_{x=b/\delta_p \text{Re} \text{Pr}}, \quad (24)$$

where $\text{Pe} = \text{Re} \text{Pr}$, b is certain characteristic length unit to be used in averaging the local mass transfer coefficient. The velocity and concentration values in formula (24)

that are the functions of x and y are obtained from a numerical solution of the system (1)–(2). The numerical solution results of the system (1)–(2) obtained using a BSEM-6 computer are approximated by the formula

$$\beta_1 = (1.37 + 2.28 \sqrt{TB}) U_0^{1/2} D^{1/2} / b^{1/2} \quad (25)$$

or

$$\beta_2 = \left[1.37 + 2.28 a_0^{1/2} \frac{\delta_p}{v^{1/2}} \left(\frac{\rho}{\sigma} \right)^{1/2} \left(\frac{h_0 \varepsilon}{gv} \right)^{1/4} \right] \frac{U_0^{1/2} D^{1/2}}{b^{1/2}}. \quad (26)$$

Note, that at $TB \rightarrow 0$ formula (26) approximates that known for the coefficient of mass transfer into a laminar liquid film. The inlet portion value according to (8) is of the form:

$$b = l_{bx} = k \delta_p \operatorname{Re}. \quad (27)$$

Substituting (27) into (26) we obtain

$$\beta_2 / D^{1/2} = \frac{Av^{1/2}}{\delta_p} + B \left(\frac{\rho}{\sigma} \right)^{1/2} \left(\frac{h_0^3 \varepsilon^3}{gv} \right)^{1/4}, \quad (28)$$

where

$$A = 1.37/k^{1/2}, \quad B = 2.28(a_0/k)^{1/2}. \quad (29)$$

In order to express the dissipation energy in terms of the mean velocity U_0 and turbulent liquid film thickness h_0 we shall use the relationship $\varepsilon = gU_0$. A number of formulas is known for approximating U_0 and h_0 in the case of turbulent conditions. Using the empirical formulas obtained in²³ relationships (26) and (28) can be reduced to the following form:

$$\beta_1 / D^{1/2} = \left[1.37 + 2.28 a_0^{1/2} \delta_p \left(\frac{\rho g}{\sigma} \right)^{1/2} \left(\frac{U_0 h_0}{v} \right)^{3/4} \right] \frac{U_0^{1/2}}{b^{1/2}} \quad (30)$$

$$\beta_2 / D^{1/2} = A \frac{v^{1/2}}{\delta_p} + B \left(\frac{\rho}{\sigma} \right)^{1/2} (gv)^{1/2} \left(\frac{U_0 h_0}{v} \right)^{3/4}, \quad (31)$$

where A and B are as before given by formula (29). From formulas (30) and (31) it follows that the mass transfer coefficient is inversely proportional to σ . We have derived theoretically the same relationship in ref.²⁴ for the liquid film wavy down-flow condition.

We shall now calculate the mass transfer coefficient without any regard for the inlet portion and also under the condition that a distributed substance concentration

does not depend on the longitudinal coordinate that is valid for the infinitely long pipe. In this case the convective diffusion equation takes the form:

$$\frac{d}{dy} [(1 + TD(y - y^2))] \frac{dc}{dy} = 0, \quad (32)$$

where

$$TB = a_1 \delta_p / v; \quad a_1 = a_{01} \left(\frac{\rho}{\sigma} \right) \left(\frac{h_0^3 e^3}{g v} \right)^{1/2}. \quad (33)$$

Therefore relationship (33) is of the same form as at the inlet portion, but the proportionality factor may be different. The boundary conditions will be selected as follows:

$$c = C_p \quad \text{at} \quad y = 0 \quad (\text{on the surface}) \quad (34)$$

$$c = C_1 \quad \text{at} \quad y = 1 \quad (\text{at the wall}) \quad (35)$$

$$c = C_1 \quad \text{at} \quad y \rightarrow \infty \quad (\text{far from the surface}) \quad (36)$$

The solution of Eq. (32) with boundary conditions (34) and (35) means that the diffusing substance is distributed over the entire film thickness and under boundary

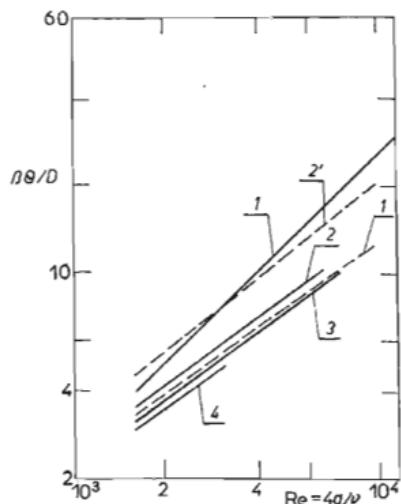


FIG. 3

Dependence of $\beta\theta/D$ on $Re = 4q/v$ for the Experimental Data of Various Authors:
1 (ref.²⁰); 2 (ref.²²); 3 (ref.¹⁸); 4 (ref.¹⁹).

Dotted lines — calculated from equations.
(1' (40), 2' (3), for $b = 1$).

conditions (34) and (36) it means that the variation in concentration occurs only within the diffusive layer adjacent to the phase interface. The calculation performed shows that final results are in practice the same and have the form:

$$\beta_3 = A_1 \left(\frac{\rho}{\sigma} \right)^{1/2} (gv)^{1/2} \left(\frac{h_0 \varepsilon}{vg} \right)^{3/4} D^{1/2} \quad (37)$$

or expressing the dissipation energy in terms of the mean velocity, we obtain:

$$\beta_3 = A_1 \left(\frac{\rho}{\sigma} \right)^{1/2} (gv)^{1/2} \left(\frac{U_0 h_0}{v} \right)^{3/4} D^{1/2}, \quad (38)$$

where $A_1 = 0.63 a_{01}$.

Formulas (30), (31) and (38) contain the constant coefficients which have been determined on the basis of the experimental data^{18,22}. Their numerical values are:

$$a_0^{1/2} = 0.055; \quad k^{1/2} = 1.9; \quad a_{01}^{1/2} = 2.5 a_0^{1/2}. \quad (39)$$

If the inlet portion length calculated by formula (27) with allowance for (39) is less than the tube length l , the mean mass transfer coefficient should be calculated by the formula:

$$\beta_{cp} = \beta_2 \frac{l_{bx}}{l} + \beta_3 \frac{l - l_{bx}}{l}, \quad (40)$$

where β_2 and β_3 are defined by formulas (31) and (38) with allowance for the coefficients of (39). Fig. 3 shows the mass transfer coefficients as calculated by formula (40); Curve 1' is for the tube of $l = 600$ cm (ref.²²). As it follows from Fig. 3, there is a satisfactory agreement between the theory and experiment. If the initial portion length under the film flow condition in question is larger than tube length, then the latter becomes to be the characteristic linear dimension. For this case the calculation of the mass transfer coefficient by formula (30) with regard for a_0 from relationship (39) is given in Fig. 3 (curve 2'). From this figure it is clear that the theory satisfactorily agrees with the experimental data²⁰.

For the turbulent flow the value δ_p in formulas (28), (29) and (31) was equal to the ultimate film thickness corresponding to the transition to turbulent regime. We took the value $\delta_p = 0.053$ cm that corresponded to $Re = 4g/v = 1500$.

REFERENCES

1. Bruley D. F.: *AICHE J.* **11**, 945 (1965).
2. Jones W. P., Launder B. E.: *Int. J. Heat Mass Transfer* **15**, 301 (1972).

3. Whitaker S., Serro R.: *Chem. Eng. Sci.* **29**, 963 (1974).
4. Serro R., Whitaker S.: *Chem. Eng. Sci.* **26**, 785 (1971).
5. Hassan N. A.: *J. Appl. Mech.* **34**, 535 (1967).
6. Haugen R.: *J. Appl. Mech.* **35**, 631 (1968).
7. Vorontsov E. G., Yakhno O. M.: *Prikl. Mat. Tekh. Fiz.*, N 4 (1974).
8. Murty N., Sastr W.: *Chem. Eng. Sci.* **28**, 869 (1973).
9. Scriven L., Pigford R.: *AICHE J.* **4**, 382 (1958).
10. Whitaker S.: *AICHE J.* **17**, 997 (1971).
11. Vilmar T.: *AICHE J.* **17**, 997 (1971).
12. Stücheli A., Örisik M. N.: *Chem. Eng. Sci.* **31**, 369 (1976).
13. *Investigation into Turbulent Flows of Two-Phase Media* (S. S. Kutateladze, Ed.) Nauka, Novosibirsk 1973.
14. Kholpanov L. P., Shkadov V. Ya., Malusov V. A., Zhavoronkov N. M.: *Teor. Osn. Khim. Tekhnol.* **10**, 659 (1976).
15. Kholpanov L. P., Malyusov V., Zhavoronkov N. M.: *Teor. Osn. Khim. Tekhnol.* **12**, 438 (1978).
16. Prasher B. D., Fricke A. L.: *Ind. Eng. Chem.* **13**, 336 (1974).
17. Zhavoronkov N. M., Malyusov V. A., Malafeiev N. A.: *Khim. Prom. (Moscow)* No 8, 240 (1951).
18. Kamei S., Oishi L.: *Mem. Fac. Eng. Kyoto Univ.* **17**, 277 (1955).
19. Borisov G. S.: *Thesis*. Moscow. Khim. Tekhnol. Inst., Moscow 1964.
20. Kulov N. N., Malyusov V. A.: *Teor. Osn. Khim. Tekhnol.* **1**, 213 (1967).
21. Emmert R. E., Pigford R. L.: *Chem. Eng. Progr.* **50**, 87 (1954).
22. Zamourelle A. P., Sandale O. C.: *Chem. Eng. Sci.* **27**, 1035 (1972).
23. Brauner H.: *V. D. I. Forschungsh.* **457** (1956).
24. Kholpanov L. P., Shkadov V. Ya., Malyusov V. A., Zhavoronkov N. M.: *Teor. Osn. Khim. Tekhnol.* **1**, 73 (1967).